Isocurvature Perturbations and Non-Gaussianities from Nonthermal Dark Matter

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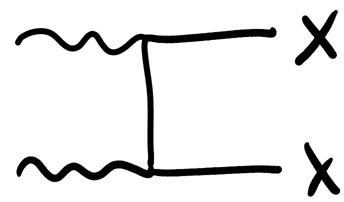
[focus: nearly completed work with Hojin Yoo]

Thermally Decoupled DM X

1) Either very massive

$$\left(\frac{200\,\mathrm{TeV}}{M_X}\right)^2 \left(\frac{T_*}{M_X}\right) < 1$$

→ never thermalizes



and/or very weakly interacting.

2) Long lived: e.g.

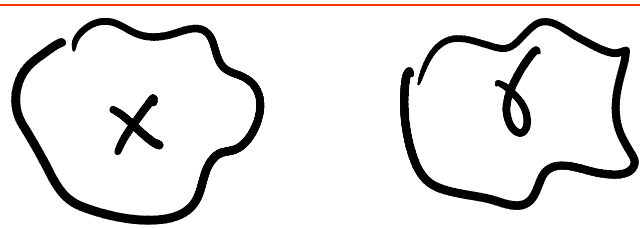
a) gauge and accidental symmetries

b) extra dimension warping

3) In the hidden sector w.r.t. the inflaton sector.

Such situations form a large class of BSM speculations.

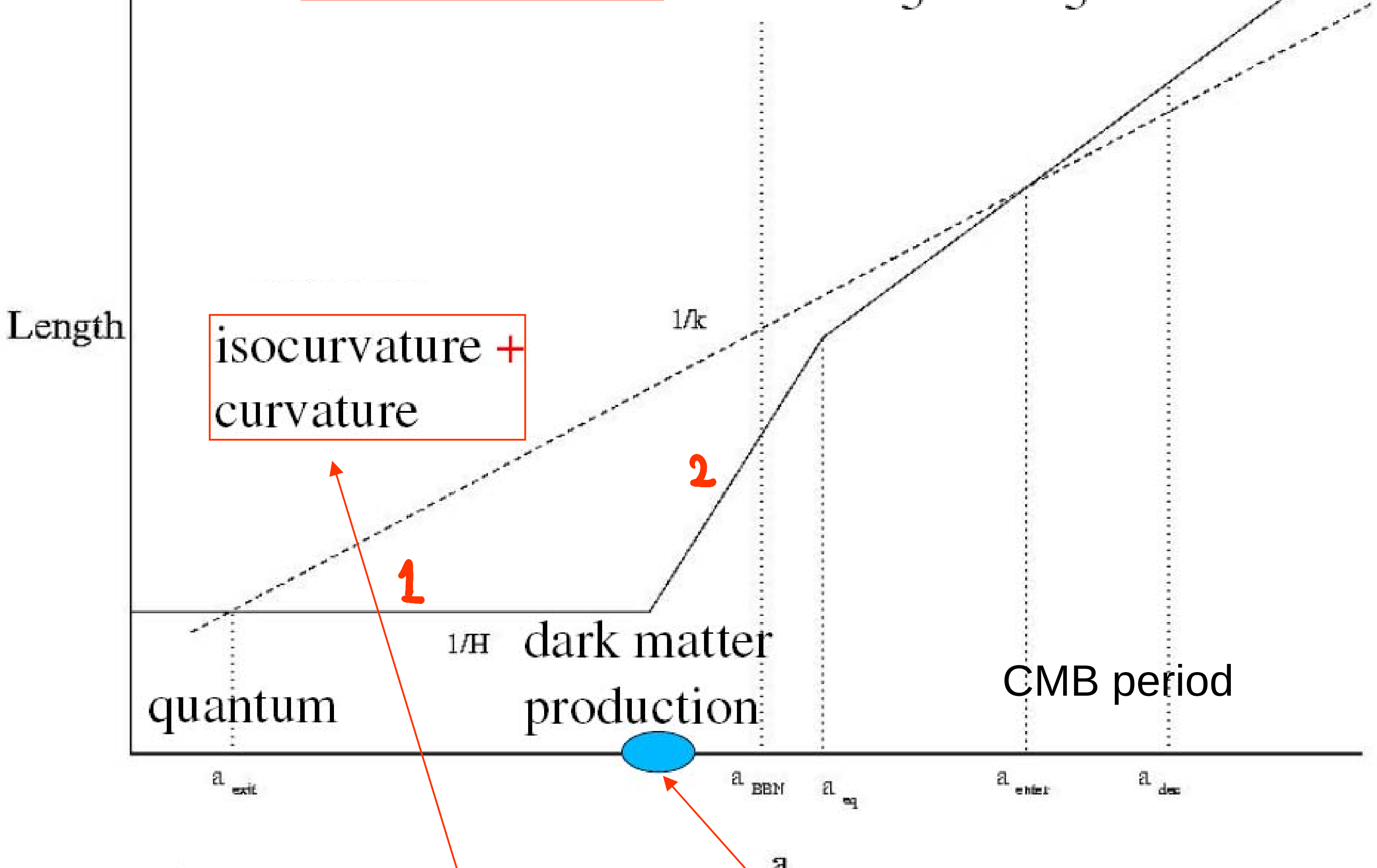
→



In this talk, focus on situation in which X is a minimally coupled scalar.

X has implications for inflationary obs.

N-point functions of $\Phi|_i = \frac{-1}{5}\delta_S - \frac{3}{5}\zeta$



Same source for both!

$\rho_X(t_e) \sim 10^{-3} m_X (H_e m_X)^{3/2} e^{-2 m_X / H_e}$

Corollary to D. Chung, E. Kolb, A. Riotto, and L. Senatore. Isocurvature constraints on gravitationally produced superheavy dark matter. *Phys. Rev. D*, Jan 2005.

Variables of Interest

$$\bar{g}_{ij} = a^2(t)\delta_{ij}$$

$$\begin{aligned} h_{00} &= -E, \\ h_{i0} &= h_{0i} = a\frac{\partial F}{\partial x^i}, \\ h_{ij} &= a^2\left[A\delta_{ij} + \frac{\partial^2 B}{\partial x^i \partial x^j}\right] \end{aligned}$$

Inflaton = curvature : $\zeta \equiv \frac{A}{2} - H\frac{\delta\rho_\phi}{\dot{\bar{\rho}}_\phi}$ → Inherited by photons

dark matter density pert. $\zeta_X \equiv \frac{A}{2} - H\frac{\delta\rho_X}{\dot{\bar{\rho}}_X}$ → $\delta_X \equiv \frac{\delta\rho_X}{\bar{\rho}_X}$

Isocurvature density pert. $\delta_S \equiv 3(\zeta_X - \zeta)$ → dark matter - photon

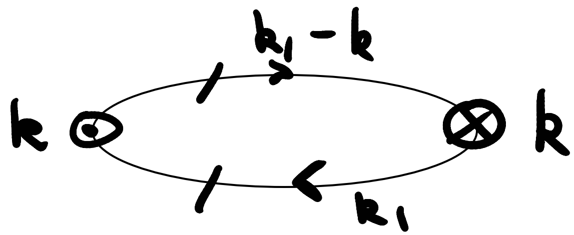
Key property distinguishing X and inflaton:

$$\delta\rho_X = \delta T_{00}^{(X)} \propto O_1 X O_1 X$$
$$\delta\rho_\phi = \delta T_{00}^{(\phi)} \propto O_2 \delta\phi$$

Quadratic!

2-Point

$$\delta\rho_X(x) \propto X^2$$



0[th] order in non-grav coup
resummed grav quadratic

$$\mathscr{P}_{\delta_X}(k) = \frac{k^3}{2\pi^2}\int d^3r e^{-i\vec{k}\cdot\vec{r}}\langle \delta_X(t,\vec{r})\delta_X(t,0)\rangle$$

$$= \frac{k^3}{2\pi^2}2\int_{k<a_e H_e}\frac{d^3k_1}{(2\pi)^3}P_X(k_1)P_X(|\mathbf{k}-\mathbf{k_1}|)$$

$$P_X(k) \equiv \frac{m_X^2}{2\rho_X}|X_k|^2 \propto \langle X(\widetilde{x)X}(y)\rangle$$

$$= 2\pi^2 A_X\left(\frac{k}{k_0}\right)^{n_X-3}$$

Minimal coupling and conformal dimension

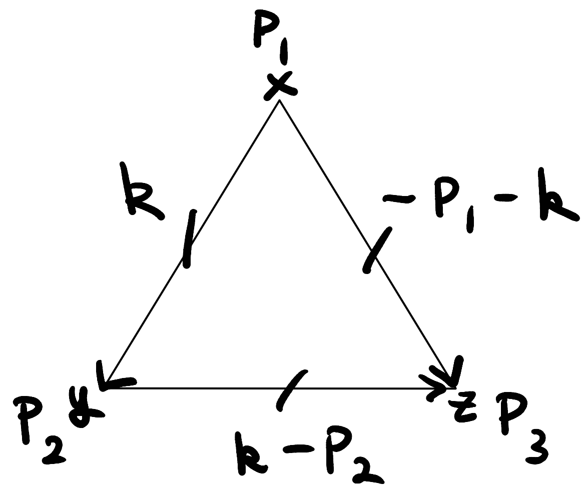
$$n_X = 4 - \sqrt{9 - \frac{4m_X^2}{H_*^2}}$$

$$A_X = \frac{10^2\cdot 2^{-4+2\nu_*}|\Gamma(\nu_*)|^2}{\pi^3}\frac{m_X H_*^2}{H_e^3}$$

$$\times\exp\left[\frac{2}{\epsilon}\Re\left\{\nu_e - \frac{3}{2}\log\left[\left(\nu_e + \frac{3}{2}\frac{H_e}{m_X}\right)\right] - \nu_* + \frac{3}{2}\log\left[\left(\nu_* + \frac{3}{2}\frac{H_*}{m_X}\right)\right]\right\}\right]$$

$$\nu = \sqrt{9/4 - m_X^2/H^2}$$

3-Point



$$B_{\delta_X}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \approx 8 \int \frac{d^3k}{(2\pi)^3} P_X(|\vec{k}|) P_X(|\vec{p}_1 + \vec{k}|) P_X(|\vec{p}_2 - \vec{k}|)$$

$$\approx \frac{8}{n_X - 1} \frac{p_{\min}^3}{2\pi^2} P_X(p_{\min}) \left[P_X(p_1) P_X(p_2) + P_X(p_2) P_X(p_3) + P_X(p_3) P_X(p_1) \right]$$

$$n_X = 4 - \sqrt{9 - \frac{4m_X^2}{H_*^2}}$$

Not X power spec

larger than P_S

$$B_{\delta_X}(\vec{p}_1,\vec{p}_2,\vec{p}_3) \approx \frac{8}{n_X-1}\frac{p_{\rm min}^3}{2\pi^2}P_X(p_{\rm min})\left[P_X(p_1)P_X(p_2)+P_X(p_2)P_X(p_3)+P_X(p_3)P_X(p_1)\right]$$

$$B_\zeta(\vec{p}_1,\vec{p}_2,\vec{p}_3) = \frac{6}{5}f_{NL}^{\rm eff}\left[P_\zeta(p_1)P_\zeta(p_2)+P_\zeta(p_2)P_\zeta(p_3)+P_\zeta(p_3)P_\zeta(p_1)\right]$$

$$\longrightarrow \boxed{f_{NL}^{\rm eff} \approx \frac{20}{3}\frac{1}{n_X-1}\left(\frac{p_{\rm min}^3}{2\pi^2}P_X(p_{\rm min})\right)\frac{P_X(p_1)P_X(p_2)+P_X(p_2)P_X(p_3)+P_X(p_3)P_X(p_1)}{P_\zeta(p_1)P_\zeta(p_2)+P_\zeta(p_2)P_\zeta(p_3)+P_\zeta(p_3)P_\zeta(p_1)}}$$

Express P_X as a function of P_ζ through the following manipulations.

Due to quadratic nature of $\delta\rho_X(x) \propto X^2$,

$$\mathscr{P}_{\delta_X}(k) \propto \frac{4}{n_X-1}P_X^2(k)$$

Isocurvature power has to be subdominant:

$$\mathscr{P}_{\delta_X} \equiv \alpha\mathscr{P}_\zeta \sim \alpha 10^{-9}$$

$$\boxed{10^{-9} \ll} \; \alpha \ll 1$$

Recall: fixed by $\frac{m_X}{H_e}$, $\frac{H_*}{H_e}$

$$\longrightarrow \boxed{\frac{P_X}{P_\zeta} \sim \frac{1}{2}\sqrt{\frac{(n_X-1)\alpha}{\mathscr{P}_\zeta}} \sim \frac{1}{2}\sqrt{\alpha(n_X-1)}10^{4.5}}$$

$$B_{\delta_X}(\vec{p}_1,\vec{p}_2,\vec{p}_3) \approx \frac{8}{n_X-1}\frac{p_{\min}^3}{2\pi^2}P_X(p_{\min})\left[P_X(p_1)P_X(p_2)+P_X(p_2)P_X(p_3)+P_X(p_3)P_X(p_1)\right]$$

$$B_\zeta(\vec{p}_1,\vec{p}_2,\vec{p}_3) = \frac{6}{5}f_{NL}^{\rm eff}\left[P_\zeta(p_1)P_\zeta(p_2)+P_\zeta(p_2)P_\zeta(p_3)+P_\zeta(p_3)P_\zeta(p_1)\right]$$

$\sqrt{\alpha(n_x-1)}\ 10^{-4.5}$ (scale dep) $\alpha(n_x-1)\ 10^8$

$$\longrightarrow\quad f_{NL}^{\rm eff} \approx \frac{20}{3}\frac{1}{n_X-1}\left(\frac{p_{\min}^3}{2\pi^2}P_X(p_{\min})\right)\frac{P_X(p_1)P_X(p_2)+P_X(p_2)P_X(p_3)+P_X(p_3)P_X(p_1)}{P_\zeta(p_1)P_\zeta(p_2)+P_\zeta(p_2)P_\zeta(p_3)+P_\zeta(p_3)P_\zeta(p_1)}$$

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Isocurvature power has to be subdominant:

$$\mathscr{P}_{\delta_X} \equiv \alpha\mathscr{P}_\zeta \sim \alpha 10^{-9} \qquad 10^{-9} \ll \alpha \ll 1$$

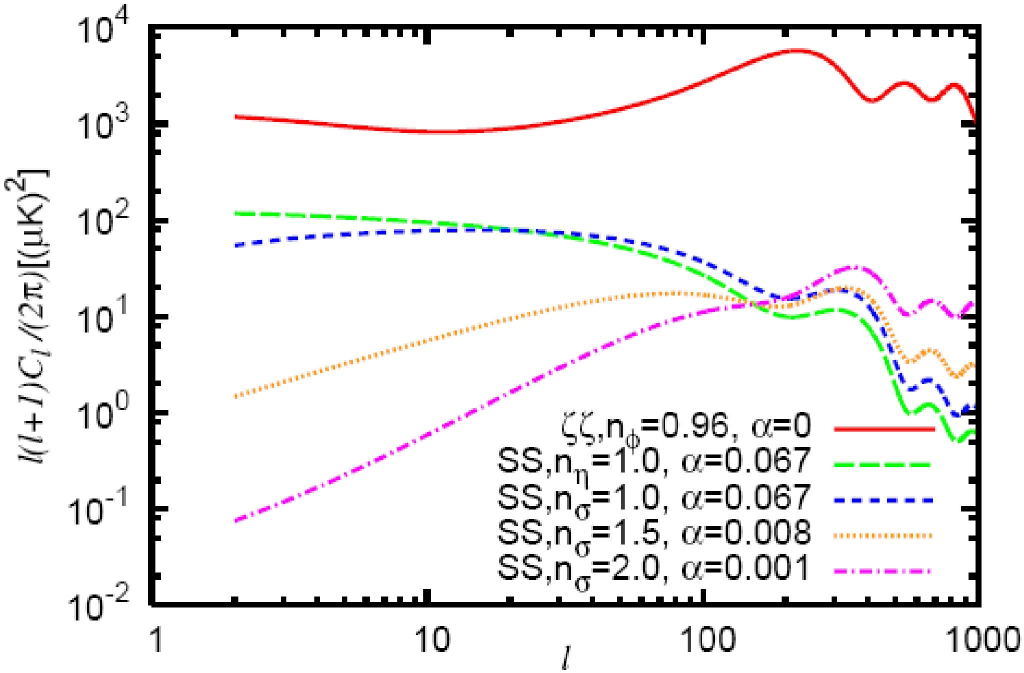
$$\longrightarrow\quad \frac{P_X}{P_\zeta} \sim \frac{1}{2}\sqrt{\frac{(n_X-1)\alpha}{\mathscr{P}_\zeta}} \sim \frac{1}{2}\sqrt{\alpha(n_X-1)}10^{4.5}$$

Depends on α which is constrained by data.

Experimental Constraint

Komatsu et al 08, 10;

C. Hikage, K. Koyama, T. Matsubara, T. Takahashi and M. Yamaguchi, *Limits on isocurvature perturbations from non-Gaussianity in WMAP temperature anisotropy* Mon. Not. Roy. Astron. Soc. **398**, 2188 (2009)



If $\langle \hat{\zeta}\hat{\delta}_S \rangle = 0$

$$\begin{aligned} \alpha &< 0.070 \quad (n_\sigma = 1) \\ \alpha &< 0.042 \quad (n_\sigma = 1.5) \end{aligned}$$

$$\alpha \equiv \frac{\mathscr{P}_{\delta_S}}{\mathscr{P}_{\delta_\zeta} + \mathscr{P}_{\delta_S}}$$

If $\langle \hat{\zeta}\hat{\delta}_S \rangle \sim -\langle \hat{\zeta}\hat{\zeta} \rangle \longrightarrow \alpha < 0.004$

Max size of α determined by corr type

No cross-correlation

- Establish non-cross-correlation in a gauge invariant manner

Decoupling between long wavelength metric pert. and particle production eq. → $\ddot{X} + 3H\dot{X} - (1-2\zeta)\frac{\nabla^2}{a^2}X + m_X^2 X = 0$

$$\hat{\delta}_s = -3H \frac{\hat{\rho}_X - \hat{1}\langle\hat{\rho}_X\rangle_{\rm ren}}{\frac{d}{dt}\langle\hat{\rho}_X\rangle_{h=0,{\rm ren}}}$$

→ $\langle\hat{\zeta}\hat{\delta}_S\rangle \ll \langle\hat{\zeta}\hat{\zeta}\rangle$ Uncorrelated type.

Explicit Numerical Example

- Compute $V(\phi) = \frac{1}{2} m_\phi^2 \phi^2$ model numerically

i.e. Compute the correlator modes numerically within this background.

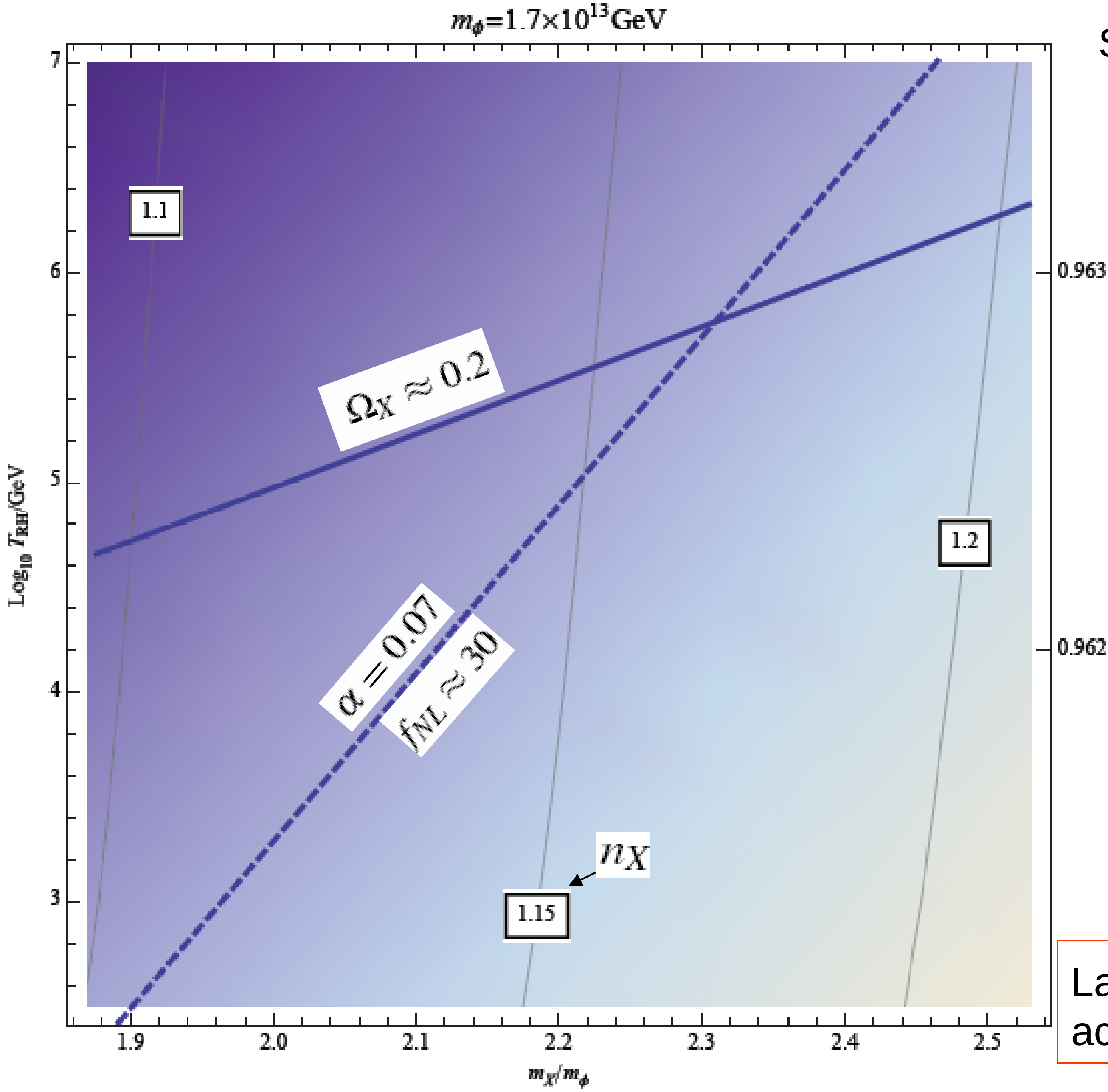
- effectively 2-parameter model:

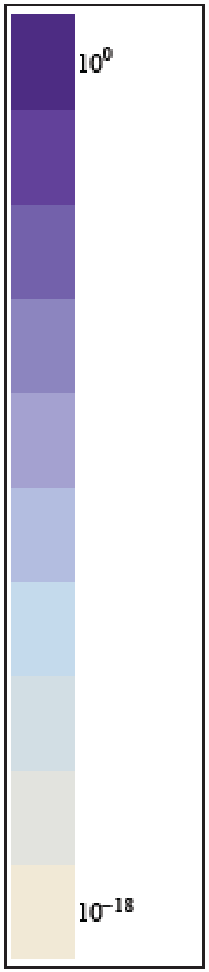
m_X controls isocurvature amplitude and DM density

T_{RH} controls k_{phys} entering isocurvature amplitude

- Generalize to mixed dark matter scenarios: e.g. $\mathscr{P}_{\delta_S} = \left(\frac{\Omega_X}{\Omega_{\mathrm{cdm}}}\right)^2 \mathscr{P}_{\delta_X}$

Squeezed triangle





Large NG is easily achieved for $m_X \lesssim 4H_e$

Summary

- Gravitationally produced nonthermal DM X is of uncorrelated type: $\langle \hat{\zeta} \hat{\delta}_S \rangle = 0$

  → sizable X isocurvature allowed by data

- Large bispectrum is possible, including the squeezed triangle limits: $f_{NL}^{\rm eff} \lesssim 30$

$m_X \lesssim 4H_e$ ← Unobservable isocurvature for $m_X \gtrsim 4H_e$

$T_{\rm RH} \lesssim 10^6 \quad {\rm GeV}$

The largeness of bispectrum is connected with $\delta\rho_X(x) \propto X^2$ and smallness of $\mathscr{P}_\zeta \sim 10^{-9}$

$$\rightarrow \frac{P_X}{P_\zeta} \sim \frac{1}{2}\sqrt{\frac{(n_X - 1)\alpha}{\mathscr{P}_\zeta}} \sim \frac{1}{2}\sqrt{\alpha(n_X - 1)}\,10^{4.5}$$